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ON THE ONSET OF THREE-DIMENSIONALITY AND TIME-DEPENDENCE IN THE GÖRTLER VORTEX PROBLEM

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ABSTRACT

The instability of large amplitude Görtler vortices in a growing boundary layer is discussed in the fully nonlinear regime. It is shown that a three-dimensional breakdown to a flow with wavy vortex boundaries similar to that which occurs in the Taylor vortex problem takes place. However, the instability is confined to the thin shear layers which were shown by Hall and Lakin (1987) to trap the region of vortex activity. The disturbance eigenfunctions decay exponentially away from the center of these layers so that the upper and lower shear layers can support independent modes of instability. The structure of the instability, in particular its location and speed of downstream propagation, is found to be entirely consistent with recent experimental results. Furthermore, it is shown that the upper and lower layers support wavy vortex instabilities with quite different frequencies. This result is again consistent with the available experimental observations.

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1. INTRODUCTION

Our concern is with the nature of the three-dimensional breakdown of steady, spanwise periodic large amplitude Görtler vortices. It is known from the experiments of Bippes and Görtler (1972) and Aihara and Koyama (1981) that this breakdown leads to a time-periodic flow with wavy vortex boundaries similar to those which occur in the Taylor problem. More recently Kohama (1987) has investigated vortex instabilities in boundary layer flows over a laminar flow wing and found a secondary instability of Görtler vortices localized at the top of the region of vortex activity. Furthermore Kohama found that the instability propagated downstream with a speed which approached the free stream speed as it developed in the downstream direction. The onset of this time-dependent motion was found in all of the above experiments to be ultimately followed by transition to turbulence. At this stage more obvious differences between the Görtler and Taylor problems emerge so that, for example, the rich bifurcation structure of the Taylor problem is apparently not carried over to the Görtler problem.

In fact even in the linear regime the apparent similarities between Taylor and Görtler vortices are perhaps misleading since it is known from the work of Hall (1982a,b, 1983, 1984) that nonparallel effects in the Görtler problem cannot in general be ignored. Indeed it was shown by Hall (1983) that the inconsistences of the various parallel-flow theories of for example Görtler (1940), Hämmerlin (1956) and later authors are a direct consequence of the parallel-flow approximation. The only regime where this difficulty with the parallel-flow theories does not occur is at small vortex wavelengths where the effect of boundary layer growth on the vortices becomes less important. However, the most surprising feature of the nonparallel theory of Hall (1983) is that the concept of a unique neutral wave is not tenable in the Görtler problem since the downstream position where a vortex begins to grow is a function of the location and initial form of the imposed disturbance. A more significant consequence of the nonparallel theory is that the concept of a unique growth rate at a given downstream location is also not tenable, this result makes transition prediction by empirical methods such as the e^n rule not possible. Nevertheless, there is much work still being done in the context of "parallel-flow" Görtler vortices.

The nonuniqueness properties of solutions of the correct zeroth order approximations to the linear Görtler vortex equations were shown by Hall (1986) to occur in the corresponding nonlinear problem at $O(1)$ wavenumbers. In the latter paper, the development of finite amplitude Görtler vortices was investigated numerically using a finite difference discretization in the normal and chordwise directions together with a Fourier expansion in the spanwise direction. It was found that as the vortices move downstream the disturbance energy of the flow becomes concentrated in the fundamental and the mean flow correction. This is entirely consistent with the weakly nonlinear theory of Hall (1982b) which is appropriate to small wavelength vortices. However in a growing boundary layer, the "local" wavenumber of a fixed wavelength disturbance grows like the displacement thickness of the boundary layer so that in most flows any vortex will eventually enter the small wavelength regime where locally the asymptotic analysis of Hall (1982a,b) apply.

The surprising feature found by Hall (1987) in a numerical simulation of nonlinear Görtler vortices is that even at relatively low wavenumbers the disturbance organizes itself so that almost all of the energy is in the fundamental and mean flow correction. This is precisely the situation found by

Hall (1982b) in an asymptotic investigation of nonlinear Görtler vortices where it was shown that the Stuart-Watson type of description of the onset of finite amplitude motion is replaced by a "mean-field" interaction.

As is the case with all weakly nonlinear stability calculations the work of Hall (1982b) is restricted to a neighborhood of the position where a given disturbance is neutrally stable. However, it can be inferred from that calculation that a vortex of nondimensional wavenumber ε^{-1} where $0 < \varepsilon << 1$ reinforces the basic flow at zeroth order at a distance $O(\varepsilon)$ downstream of the neutral point. This result was recently developed by Hall and Lakin (1987), hereafter referred to as HL, to give an asymptotic description of fully nonlinear Görtler vortices at $O(1)$ distances beyond the neutral point. It is the instability of this type of vortex which we will investigate in this paper. However, before discussing the nature of this instability we need to point out the salient properties of the HL calculations.

Consider then a Görtler vortex of wavenumber ε^{-1} developing in a boundary layer flow with Görtler number of $O(\varepsilon^{-4})$. This choice of small wavelength vortices is not as restrictive as it might first appear since, as explained above, this regime is always approached by a fixed wavelength vortex in a growing boundary layer. Suppose further that the flow is neutrally stable at the downstream location $x = x_n$, then HL showed that for $x > x_n$ the flowfield splits up as shown in Figure 1. The vortex activity is confined to region I and decays exponentially to zero in the thin shear layers IIa,b. In regions IIIa,b there is no vortex activity and the mean flow satisfies the boundary layer equations. However, in region I the mean flow is determined as a solvability condition on the equations for the fundamental. In fact, the mean flow adjusts itself so as to make the fundamental and all the higher harmonics neutral in I. The mean flow equations then determine the vortex velocity field in I so that there is a complete reversal of the usual roles of the mean flow and harmonic equations compared to say the situation in flows where nonlinearity can be described by the Stuart-Watson method. The shear layers located at y_1 and y_2 change position as they move downstream; their positions are determined from the solution of a double free boundary problem associated with the boundary layer equations. However, in flows where the local Görtler number increases faster than the fourth power of the local wave-number HL showed that y_1 migrates to the wall whilst y_2 moves to or beyond the edge of the boundary layer. The mean downstream velocity components in the layers IIa,b then approach the free stream speed and zero respectively; this has fundamental implications for the time dependent structure of the breakdown of this flow.

We shall seek secondary instabilities of the flow in layers IIa,b; more precisely we superimpose spanwise periodic travelling waves on the flow in these layers and see how they develop. These perturbations are $\frac{\pi}{2}$ radians out of phase with the fundamental so the secondary instability if it occurs will produce locally wavy vortex boundaries in IIa,b. It is of course not obvious that, should wavy vortices occur, the regions IIa,b should be particularly susceptible to these modes. In order to see why this is the case, we consider the model equation

$$\left\{\frac{\partial^2}{\partial y^2} - y\frac{\partial}{\partial x} - \frac{\partial}{\partial t} - \frac{y^2}{4} + \lambda\right\}\psi = \psi|\psi|^2. \qquad (1.1)$$

together with the condition $\psi \to 0$, $|y| \to \infty$.

In fact, this equation essentially governs the nonlinear growth of time-dependent Görtler vortices in curved channel flows. Here λ is a parameter, t denotes time, x is the distance around the channel, and y is the distance from the center of the internal viscous layer where the vortices initially develop. A finite amplitude solution of (1.1) representing a steady, x independent vortex $\psi = \psi_v$ satisfies

$$\{\frac{\partial^2}{\partial y^2} - \frac{y^2}{4} + \lambda\}\psi_v = \psi_v^3, \tag{1.2}$$

and the instability of this flow to a travelling vortex like perturbation can be calculated by setting

$$\psi = \psi_v + \psi'(x,y,t)$$

where ψ' is a real function of x, y and t. If we now look at disturbances with $\psi' = \text{Real}(e^{ikx+\sigma t}\phi(y))$ then we find that $\phi(y)$ satisfies

$$(\frac{\partial^2}{\partial y^2} - \frac{y^2}{4} + \lambda - iky - \sigma)\phi = 3\psi_v^2\phi, \qquad \phi \to 0, \ |y| \to \infty. \tag{1.3}$$

which determines an eigenrelation $\sigma = \sigma(k)$. However if the wave is $\frac{\pi}{2}$ radians out of phase in the spanwise direction with the fundamental vortex then (1.3) becomes

$$(\frac{\partial^2}{\partial y^2} - \frac{y^2}{4} + \lambda - iky - \sigma)\phi = \psi_v^2\phi, \qquad \phi \to 0, \ |y| \to \infty. \tag{1.4}$$

The latter eigenvalue problem was studied by Shaw (1985) who found that unstable modes occur for $\lambda = O(1)$. (In contrast to this result (1.3) leads to stable disturbances.) Here λ plays the role of Görtler number so increasing λ is equivalent to increasing x for the boundary layer problem. Of more relevance to our problem is the solution of (1.4) for $\lambda >> 1$. In this case ψ_v develops a triple layer structure with a core region with $\psi_v = (\lambda - y^2/4)^{1/2}$ trapped between shear layers of thickness $\lambda^{-1/6}$ at $y = \pm(4\lambda)^{1/2}$. These shear layers correspond to IIa,b in the HL calculation. An examination of (1.4) in this limit shows that any eigenvalues must now concentrate in IIa,b because in the core ϕ now satisfies

$$\left(\frac{\partial^2}{\partial y^2} - iky - \sigma\right)\phi = 0.$$

This is an Airy equation so that $\phi \to \infty$ when $y \to 2\sqrt{\lambda}$ or $y \to -2\sqrt{\lambda}$ and matching with the corresponding solution in the shear layers cannot be achieved. Thus the eigenfunctions must now concentrate in the shear layers and decay away from the centers of the layers.

The above structure for the model equation is sufficiently close to the boundary layer problem for it to be applicable there. Thus, after formulating our instability equations in Section 2 we will in Section 3 investigate the instability of IIa,b to wavy vortex modes. In Section 4 we present the results of our calculation whilst in Section 5 we draw some conclusions.

2. FORMULATION OF THE PROBLEM

The flow under consideration is that described by HL. We consider the flow of an incompressible, viscous fluid of kinematic viscosity ν and density ρ, over a wall of variable concave curvature $a^{-1}\chi(X/L)$. Here

a is a typical radius of curvature, X denotes distance along the wall and L is a typical length scale along the wall. The Reynolds number for the flow, R_E, is defined by

$$R_E = \frac{U_0 L}{\nu}, \tag{2.1}$$

where U_0 is a typical flow velocity. A curvature parameter, δ_c, is defined by

$$\delta_c = \frac{L}{a}. \tag{2.2}$$

We are interested in the limit $R_E \to \infty$ with the Görtler number G, defined by

$$G = 2R_E^{1/2}\delta_c, \tag{2.3}$$

held fixed. We denote time by T and (X,Y,Z) are taken to be the co-ordinates along the wall, normal to the wall and in the spanwise direction respectively. If (U,V,W) denotes the corresponding velocity vector we define dimensionless co-ordinates (x,y,z) and velocity (u,v,w) by

$$(x,y,z) = L^{-1}(X, YR_E^{1/2}, ZR_E^{1/2}),$$

and

$$(u,v,w) = U_0^{-1}(U, VR_E^{1/2}, WR_E^{1/2}).$$

Our analysis is restricted to flows with $u \to 1$ when $y \to \infty$ and the pressure P is written in the form

$$P = \rho \frac{U_0^2}{R_E} p.$$

The continuity equation and non-dimensional unsteady Navier-Stokes equations for the flow take the form

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0,$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{dx} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2},$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{1}{2}G\chi u^2 - \frac{\partial p}{\partial y} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}, \qquad (2.4a,b,c,d)$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{\partial p}{\partial z} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}.$$

Here the non-dimensional time variable t is given by $t = U_0L^{-1}T$, and terms of order $R_E^{-1/2}$ have been neglected. HL obtained a steady solution of (2.4) which satisfies

$$u = v = w = 0, \qquad y = 0, \qquad (2.5a,b)$$

$$u \to 1, \qquad y \to \infty.$$

This was an asymptotic solution valid in the limit of small vortex wavelength. The Görtler number G, defined in (2.3), is expanded in the form

$$G = G_0\varepsilon^{-4} + G_1\varepsilon^{-3} + \cdots, \qquad (2.6)$$

where ε^{-1} is the non-dimensional wavenumber of the vortices. In the main part of the boundary layer $\overline{u}(x,y)$, the zeroth order, z-independent part of the downstream velocity component, satisfies

$$G_0 \chi \bar{u} \frac{\partial \bar{u}}{\partial y} = 1,$$

which reflects the fact that here the mean flow is driven by finite amplitude vortices. The velocity field of the (smaller) vortices is then found by considering the z-independent part of the equations of motion in I. This calculation shows that the vortices are trapped between y_1 and y_2 and formally decay to zero exponentially in IIa,b. Above y_2 and below y_1 there is no z-dependence to the flow and it is obtained by solving the boundary layer equations with jump conditions at y_1, y_2. The shear layers IIa,b correspond to the $\lambda^{-1/6}$ layers for the model problem and we now examine the flow structure in IIa. In fact our analysis is equally applicable to region IIb so our theory determines the stability of both shear layers which trap the vortices. In order to investigate the instability of the boundary layer in this region we consider perturbations to the steady, basic flow satisfying (2.4) and (2.5) in region IIa.

3. THE ASYMPTOTIC STRUCTURE OF THE WAVY MODES

It was shown by HL that layers IIa,b are of thickness $\varepsilon^{2/3}$ so that in IIa we write

$$\xi = \frac{(y-y_2)}{\varepsilon^{2/3}},$$

where $y_2(x)$ is the location of the layer IIa. Thus, in IIa we replace $\frac{\partial}{\partial x}$ by $\frac{\partial}{\partial x} - \frac{y_2'}{\varepsilon^{2/3}} \frac{\partial}{\partial \xi}$ and $\frac{\partial}{\partial y}$ by $\frac{1}{\varepsilon^{2/3}} \frac{\partial}{\partial \xi}$. The basic, steady expansions in IIa, satisfying (2.4) and (2.5), are given by equation (3.10) in HL and can be written in the form

$$u = u_B = \bar{\bar{u}}_0 + \varepsilon^{2/3}\,\bar{\bar{u}}_1 + \cdots + \varepsilon^{4/3}\cos(\tfrac{z}{\varepsilon})(U_{01} + \varepsilon^{2/3}U_{11} + \cdots)$$

$$+ \varepsilon^{8/3}\cos(\tfrac{2z}{\varepsilon})(U_{02} + \varepsilon^{2/3}U_{12} + \cdots) + \cdots ,$$

$$v = v_B = \bar{\bar{v}}_0 + \varepsilon^{2/3}\,\bar{\bar{v}}_1 + \cdots + \varepsilon^{-2/3}\cos(\tfrac{z}{\varepsilon})(V_{01} + \varepsilon^{2/3}V_{11} + \cdots)$$

$$+ \varepsilon^{2/3}\cos(\tfrac{2z}{\varepsilon})(V_{02} + \varepsilon^{2/3}V_{12} + \cdots) + \cdots , \qquad (3.1a,b,c,d)$$

$$w = w_B = \varepsilon^{-1/3}\sin(\tfrac{z}{\varepsilon})(W_{01} + \varepsilon^{2/3}W_{11} + \cdots) + \varepsilon^{1/3}\sin(\tfrac{2z}{\varepsilon})(W_{02} + \varepsilon^{2/3}W_{12} + \cdots) + \cdots ,$$

$$p = p_B = \bar{\bar{p}}_0 + \varepsilon^{2/3}\,\bar{\bar{p}}_1 + \cdots + \varepsilon^{-4/3}\cos(\tfrac{z}{\varepsilon})(P_{01} + \varepsilon^{2/3}P_{11} + \cdots)$$

$$+ \varepsilon^{2/3}\cos(\tfrac{2z}{\varepsilon})(P_{02} + \varepsilon^{2/3}P_{12} + \cdots) + \cdots ,$$

where the coefficients are functions of x and ξ. Note that the coefficients U_{jk}, V_{jk}, W_{jk}, P_{jk} in the above formulation correspond to $2U_{jk}$, $2V_{jk}$, $2iW_{jk}$, $2P_{jk}$ respectively, $j = 0,1,2,\cdots$, $k = 1,2,3,\cdots$, in the formulation of HL. The equations obtained from substitution of the basic expansions (3.1) into the continuity and Navier-Stokes equations (2.4) and equating coefficients of like powers of ε are given in detail by HL. The equation obtained which determines V_{01}, from (3.1b), is

$$\frac{\partial^2 V_{01}}{\partial \xi^2} + g_1 \xi V_{01} = \frac{V_{01}^3}{6} - g_2 f V_{01}, \qquad (3.2)$$

where

$$g_1(x) = \left(\frac{1}{a+2y_2} - \frac{\chi'(a+2y_2)^{3/2}}{3\chi\sqrt{G_0\chi}} - b\right)/3,$$

and

$$g_2(x) = \frac{\sqrt{G_0 \chi}\ \sqrt{a+2y_2}}{3} . \qquad (3.3a,b)$$

Here $f(x)$ is a function which can only be determined at higher order, $a(x)$ and $b(x)$ are arbitrary functions of x arising from the solution in region I (see (3.7) in HL), and a dash denotes a derivative with respect to x.

The results of Davey, DiPrima and Stuart (1968) show that the Taylor-vortex flow is unstable against perturbations differing in phase from the fundamental component of the steady vortex flow by $\frac{\pi}{2}$. After instability, the new flow has wavy surfaces traveling in the azimuth separating neighbouring vortices. This suggests that, since we are seeking a secondary instability that will produce locally wavy vortex boundaries in IIa,b, we must consider a $\frac{\pi}{2}$ out-of-phase, time-dependent perturbation to the basic flow in IIa,b.

We have also considered the case of an in-phase perturbation and found the resulting problem more complicated than the present one. We anticipate that there are no unstable solutions for this case and just proceed with the present situation.

Hence, we seek solutions with out-of-phase perturbations proportional to

$$E = \exp\left(\frac{i}{\varepsilon^2} \int^x K(x)dx - \frac{i\Omega t}{\varepsilon^2}\right), \qquad (3.4)$$

where the wavenumber K expands as

$$K = K_0 + \varepsilon^{2/3} K_1 + \cdots , \qquad (3.5)$$

and Ω is the constant frequency. The length scale and time scale in (3.4) are chosen, from Hall (1982a), so that $\frac{\partial^2}{\partial z^2} \sim \frac{\partial}{\partial x}$ and $\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} = 0$ in the shear layer, the latter scaling ensures that the waves travel downstream with the speed of the fluid in the shear layer.

We find that the appropriate expansions in IIa take the form

$$u = u_B + \{\delta(\varepsilon^{4/3}\sin(\tfrac{z}{\varepsilon})E(u_{01} + \varepsilon^{2/3}u_{11} + \cdots) + \cdots) + \cdots + c.c.\},$$

$$v = v_B + \{\delta(\varepsilon^{-2/3}\sin(\tfrac{z}{\varepsilon})E(v_{01} + \varepsilon^{2/3}v_{11} + \cdots) + \cdots) + \cdots + c.c.\},$$

(3.6a,b,c,d)

$$w = w_B + \{\delta(\varepsilon^{-1/3}\cos(\tfrac{z}{\varepsilon})E(w_{01} + \varepsilon^{2/3}w_{11} + \cdots) + \varepsilon^{-1/3}E(w_{m0} + \varepsilon^{2/3}w_{m1} + \cdots) + \cdots)$$
$$+ \cdots + c.c.\},$$

$$p = p_B + \{\delta(\varepsilon^{-4/3}\sin(\tfrac{z}{\varepsilon})E(p_{01} + \varepsilon^{2/3}p_{11} + \cdots) + \cdots) + \cdots + c.c.\},$$

where δ is a small amplitude and c.c. denotes complex conjugate. Note that there are only mean flow (independent of z) correction terms occurring in the expansion for w as a result of the perturbation being $\frac{\pi}{2}$ out of phase with the fundamental vortex. For the case of an in-phase perturbation mean flow correction terms also occur in the corresponding expansions for u, v, and p and the resulting eigenvalue problem is more complex.

The coefficents are functions of x and ξ and are determined by substitution of the expansions (3.6) into the unsteady equations of motion (2.4). The zeroth order equations obtained from equating coefficients of $\delta\sin(\frac{z}{\varepsilon})E$ in (2.4a,b,c) and $\delta\cos(\frac{z}{\varepsilon})E$ in (2.4d) are

$$\frac{\partial v_{01}}{\partial \xi} - w_{01} = 0,$$

$$-i\Omega u_{01} + iK_0\bar{\bar{u}}_0 u_{01} + v_{01}\frac{\partial \bar{\bar{u}}_1}{\partial \xi} = -u_{01},$$

$$-i\Omega v_{01} + iK_0\bar{\bar{u}}_0 v_{01} = -G_0\chi\bar{\bar{u}}_0 u_{01} - v_{01}, \qquad (3.7a,b,c,d)$$

$$0 = -p_{01} - w_{01}.$$

Consistency of (3.7b,c) requires that

$$\Omega = K_0\bar{\bar{u}}_0, \qquad (3.8)$$

which shows that the waves move downstream with mean speed $\bar{\bar{u}}_0$, i.e., the speed of the mean part of the basic flow in IIa. Thus (3.7) reduces to

$$\frac{\partial v_{01}}{\partial \xi} - w_{01} = 0,$$

$$u_{01} + \frac{\partial \bar{\bar{u}}_1}{\partial \xi} v_{01} = 0,$$

$$v_{01} + G_0\chi\bar{\bar{u}}_0 u_{01} = 0, \qquad (3.9a,b,c,d)$$

$$w_{01} + p_{01} = 0.$$

(Note from HL that $G_0\chi\bar{\bar{u}}_0\frac{\partial \bar{\bar{u}}_1}{\partial \xi} = 1$). From equating coefficients of $\delta\sin(\frac{z}{\varepsilon})E$ in (2.4b,c) at the next order we obtain the equations

$$iK_0\bar{\bar{u}}_1u_{01} + iK_1\bar{\bar{u}}_0u_{01} + \frac{\partial\bar{\bar{u}}_1}{\partial\xi}v_{11} + \frac{\partial\bar{\bar{u}}_2}{\partial\xi}v_{01} - w_{m0}U_{01} = \frac{\partial^2u_{01}}{\partial\xi^2} - u_{11},$$

$$iK_0\bar{\bar{u}}_1v_{01} + iK_1\bar{\bar{u}}_0v_{01} - w_{m0}V_{01} = -G_0\chi\bar{\bar{u}}_0u_{11} - G_0\chi\bar{\bar{u}}_1u_{01} - \frac{\partial p_{01}}{\partial\xi} + \frac{\partial^2v_{01}}{\partial\xi^2} - v_{11}. \tag{3.10a,b}$$

It is useful here to note that from (3.12) of HL

$$\bar{\bar{u}}_0 = \frac{\sqrt{a+2y_2}}{\sqrt{G_0\chi}}, \quad \bar{\bar{u}}_1 = \frac{\xi}{\sqrt{G_0\chi}\sqrt{a+2y_2}}.$$

We substitute for v_{11} from (3.10a) into (3.10b) and use the solutions for $\bar{\bar{u}}_0$ and $\bar{\bar{u}}_1$ given above to obtain

$$\frac{\partial^2v_{01}}{\partial\xi^2} + g_1\xi v_{01} = \frac{V_{01}^2v_{01}}{6} - g_2fv_{01} + \frac{2i\Omega\xi v_{01}}{3(a+2y_2)} + \frac{2iK_1\sqrt{a+2y_2}v_{01}}{3\sqrt{G_0\chi}} - \frac{2}{3}w_{m0}V_{01}. \tag{3.11}$$

Hence, in order to obtain a solution for v_{01} we require solutions for w_{m0} and V_{01}. In order to determine w_{m0} we equate coefficients of δE in (2.4d). The resulting equation is

$$i\Omega\frac{\bar{\bar{u}}_1}{\bar{\bar{u}}_0}w_{m0} + iK_1\bar{\bar{u}}_0w_{m0} + V_{01}\frac{\partial^2v_{01}}{\partial\xi^2} - v_{01}\frac{\partial^2V_{01}}{\partial\xi^2} = \frac{\partial^2w_{m0}}{\partial\xi^2}. \tag{3.12}$$

On substitution of $\frac{\partial^2v_{01}}{\partial\xi^2}$ and $\frac{\partial^2V_{01}}{\partial\xi^2}$ from (3.11) and (3.2) respectively into (3.12) we obtain

$$\frac{\partial^2w_{m0}}{\partial\xi^2} - \frac{i\Omega\xi}{a+2y_2}w_{m0} - \frac{iK_1\sqrt{a+2y_2}}{\sqrt{G_0\chi}}w_{m0} = -\frac{2}{3}w_{m0}V_{01}^2 \tag{3.13}$$

$$+ \frac{2i\Omega\xi}{3(a+2y_2)} v_{01}V_{01} + \frac{2iK_1\sqrt{a+2y_2}}{3\sqrt{G_0\chi}} v_{01}V_{01}.$$

Thus, once we have a solution for V_{01} from (3.2), it remains to solve the coupled equations (3.11) and (3.13) with the boundary conditions

$$v_{01},\ w_{m0} \to 0 \quad \text{as} \quad \xi \to \pm\infty. \tag{3.14}$$

In order to eliminate $f(x)$ from (3.2) we introduce the variable ξ_1 and let

$$\xi = \xi_1 - \frac{g_2 f}{g_1}, \tag{3.15}$$

so that (3.2) becomes

$$\frac{\partial^2 V_{01}}{\partial \xi_1^2} + g_1 \xi V_{01} = \frac{V_{01}^3}{6}. \tag{3.16}$$

If we look for a solution with K_1 of the form

$$K_1 = \tilde{K}_1 + \frac{\Omega g_2 f\sqrt{G_0\chi}}{g_1(a+2y_2)^{3/2}}, \tag{3.17}$$

then, using the transformation (3.15), equations (3.11) and (3.13) become, respectively,

$$\frac{\partial^2 v_{01}}{\partial \xi_1^2} + g_1\xi_1 v_{01} = \frac{V_{01}^2 v_{01}}{6} + \frac{2i\Omega\xi_1 v_{01}}{3(a+2y_2)} + \frac{2i\tilde{K}_1\sqrt{a+2y_2}\ v_{01}}{3\sqrt{G_0\chi}} - \frac{2}{3} w_{m0}V_{01}, \tag{3.18}$$

and

$$\frac{\partial^2 w_{m0}}{\partial \xi_1^2} - \frac{i\Omega\xi_1 w_{m0}}{a+2y_2} - \frac{i\tilde{K}_1\sqrt{a+2y_2}\ w_{m0}}{\sqrt{G_0\chi}} = -\frac{2}{3} w_{m0}V_{01}^2 \tag{3.19}$$

$$+ \frac{2i\Omega\xi_1 v_{01} V_{01}}{3(a+2y_2)} + \frac{2i\tilde{K}_1\sqrt{a+2y_2}\, v_{01}V_{01}}{3\sqrt{G_0\chi}} .$$

In order to simplify (3.16) further we make the transformations

$$\eta = (-g_1)^{1/3}\xi_1, \tag{3.20}$$

and

$$V_{01} = \sqrt{6}(-g_1)^{1/3}V_n(\eta), \tag{3.21}$$

with the result that (3.16) becomes

$$\frac{d^2V_n}{d\eta^2} - \eta V_n = V_n^3 . \tag{3.22}$$

Note that $g_1(x)$, defined by (3.3a) is less than zero for all values of x. As noted by HL, (3.22) is a particular form of the second Painleve transcendent and has been shown by Hastings and Mcleod (1978) to have a solution such that

$$V_n \sim (-\eta)^{1/2}, \ \eta \to -\infty,$$

and (3.23a,b)

$$V_n \sim \sqrt{2}\ \mathrm{Ai}(\eta), \ \eta \to +\infty.$$

A solution for V_n was obtained numerically by using (3.23) and starting integrating at $\eta = +\infty$ and integrating to the left with a fourth-order Runge-Kutta scheme. These results were used to determine solutions for v_{01} and w_{m0}.

For a fixed, real value of Ω we can solve (3.18) and (3.19) and determine the complex function $\tilde{K}_1(x)$ as x moves downstream. However, we are interested in neutrally stable solutions so we seek real values of Ω and $\tilde{K}_1(x)$. We can eliminate the x-dependence from the coefficients in (3.18) and (3.19) by making the transformations

$$v_{01} = (-g_1)^{-1/3} V_p(\eta), \tag{3.24}$$

$$\tilde{K}_1 = \frac{\hat{K}_1\sqrt{G_0 x}\;(-g_1)^{2/3}}{\sqrt{a+2y_2}}, \tag{3.25}$$

and

$$\Omega = \hat{\Omega}(-g_1)(a+2y_2). \tag{3.26}$$

Now we can find the constants $\hat{K}_1$ and $\hat{\Omega}$ so that the flow is neutrally stable at the location x where $\tilde{K}_1$ and Ω satisfy (3.25) and (3.26). Hence, with the transformations (3.2a), (3.24), (3.25) and (3.26), equations (3.18) and (3.19) become, respectively,

$$\frac{d^2V_p}{d\eta^2} - (1 + \frac{2i\hat{\Omega}}{3})\eta V_p - \frac{2i\hat{K}_1}{3} V_p = V_n^2 V_p - \frac{2\sqrt{6}}{3} w_{m0} V_n,$$

$$\frac{d^2 w_{m0}}{d\eta^2} - i\hat{\Omega}\eta w_{m0} - i\hat{K}_1 w_{m0} = 4w_{m0}V_n^2 + \frac{2\sqrt{6}}{3} i\hat{\Omega}\eta V_p V_n + \frac{2\sqrt{6}}{3} i\hat{K}_1 V_p V_n. \tag{3.27a,b}$$

We seek solutions of (3.27), with $\hat{K}_1$ and $\hat{\Omega}$ real, satisfying

$$V_p,\; w_{m0} \to 0 \quad \text{as} \quad \eta \to \pm\infty. \tag{3.28}$$

As $\eta \to +\infty$ (3.27) can be written as

$$\frac{d^2V_p}{d\eta^2} - (1 + \frac{2i\hat{\Omega}}{3})\eta V_p - \frac{2i\hat{K}_1}{3} V_p = 0,$$

and

$$\frac{d^2 w_{m0}}{d\eta^2} - i\hat{\Omega}\eta w_{m0} - i\hat{K}_1 w_{m0} = 0. \qquad (3.29a,b)$$

Hence, as $\eta \to +\infty$ we can find two independent solutions for V_p and w_{m0}, in terms of the Airy function Ai, which satisfy (3.28). When $\eta \to -\infty$ the equations for V_p and w_{m0} are

$$\frac{d^2 V_p}{d\eta^2} - \frac{2i\hat{\Omega}}{3}\eta V_p - \frac{2i\hat{K}_1 V_p}{3} = -\frac{2\sqrt{6}}{3} w_{m0}\sqrt{-\eta} \quad ,$$

$$\frac{d^2 w_{m0}}{d\eta^2} - (i\hat{\Omega} + 4)\eta\, w_{m0} - i\hat{K}_1 w_{m0} = \frac{2\sqrt{6}}{3} i\hat{\Omega}\eta\sqrt{-\eta}\, V_p + \frac{2\sqrt{6}}{3} i\hat{K}_1 V_p\sqrt{-\eta} \, . \qquad (3.30a,b)$$

The appropriate expansions of (3.30) now take the form

$$V_p = e^{-\phi|\eta|^{3/2}}[V_{p0} + \cdots],$$

$$w_{m0} = |\eta|^{1/2} e^{-\phi|\eta|^{3/2}}[w_{m00} + \cdots],$$

where ϕ satisfies

$$\phi^4 + \phi^2[4 + \frac{5i}{3}\hat{\Omega}] + (8 + i\hat{\Omega})\frac{2i\hat{\Omega}}{3} = 0,$$

and we take the two roots of this equation with positive real part to generate two independent solutions of (3.30) with V_p, $w_{m0} \to 0$, $\eta \to -\infty$.

These asymptotic solutions for V_p and w_{m0} at $\eta = \pm\infty$ were used as initial values in the numerical integration scheme used to solve (3.27). Equations (3.27) were written as a system of four first order differential equations. This system was solved using a standard fourth order Runge-Kutta integration scheme. The integration procedure was started at $\eta = -\infty$ and $\eta = +\infty$ and continued to $\eta = 0$, finding two independent solutions for V_p and w_{m0} from each direction. At $\eta = 0$ the continuity of a linear combination of the independent solutions from each direction produces an eigenvalue problem for $\hat{K}_1$ and $\hat{\Omega}$. We used a Newton-Rahpson iteration scheme for two variables to find real values for $\hat{K}_1$ and $\hat{\Omega}$. Using the above scheme solutions for $\hat{K}_1$, $\hat{\Omega}$, V_p and w_{m0} were obtained but we postpone the discussion of these results until the next section.

Having found $\hat{\Omega}$ and $\hat{K}_1$ the dimensionless frequency and wavenumber, Ω and K respectively, can only be found once the HL calculation has been performed for a particular curvature distribution $\chi(x)$. However HL gave asymptotic solutions of the free boundary problem for x close to the linear neutral position, $x = x^*$, and for x a long way downstream of that position for curvature distributions which increase as quickly as $x^{1/2}$ for $x \gg 1$.

Firstly, we recall that when $x \to x_+^*$ the shear layers coalesce. Thus if we denote by Ω_T and Ω_L the frequencies of the wavy vortex modes neutral in the upper and lower layers at x it follows that

$$\frac{\Omega_L}{\Omega_T} \to 1, \ x \to x_+^* \ . \tag{3.31}$$

Next suppose that $\chi \sim x^M$, $M > \frac{1}{2}$ for large x, the asymptotic forms for a, y_1, y_2 and b are all given in HL so tht g_{1T}, g_{1L} the values of g_1 are given by

$$g_{1T} \sim \frac{M}{9} x^{M-1} G_0, \ g_{1L} \sim \frac{M^2}{27G_0} x^{3M-2}. \tag{3.32}$$

(We note here in passing that g_1 in the lower layer is positive.) Thus for $x >> 1$ we obtain

$$\Omega_T \sim \hat{\Omega} \frac{M}{9} x^{2M-1} G_0^2, \ \Omega_L \sim \frac{\hat{\Omega}}{3}$$

or

$$\frac{\Omega_T}{\Omega_L} = \frac{MG_0^2}{3} x^{2M-1}.$$

It follows that as x increases the frequency of the upper layer mode which is neutral at x increases whilst that of the lower layer tends to a constant value. Thus we can distinguish between the modes as being of high and low frequency respectively; this result is entirely consistent with the experimental results discussed in the next section.

4. RESULTS AND DISCUSSION

The numerical scheme outlined above was used to search for eigenvalues $(\hat{K}_1,\hat{\Omega})$ in the region $\hat{K}_1 > 0$, $\hat{\Omega} > 0$. The only eigenvalues located were

$$(\hat{K}_1,\hat{\Omega}) = (0.97,0.58), \ (\hat{K}_1,\hat{\Omega}) = (5.132,1.805). \qquad (4.1a,b)$$

It is possible that other eigenvalues exist at higher values of $\hat{K}_1$, $\hat{\Omega}$ since for $|\hat{K}_1|$, $|\hat{\Omega}| > 0$ a detailed eigenvalue search was not carried out. For any particular incoming boundary layer profile the frequency Ω and wavenumber $K = K_0 + \varepsilon^{2/3}K_1$ are then calculated from (3.5), (3.17), (3.25), (3.26) and (4.1). This can only be done once the HL calculation has been carried out and the frequency and wavenumber obtained in this way will of course be dependent on x. The resulting expressions for Ω and K should be interpreted as the frequency and wavenumber which are neutrally stable at x. Alternatively we could invert the equation

$$\Omega = \Omega(x,\varepsilon),$$

to find the downstream location where the wavy vortex is neutrally stable. If Ω is held fixed at the neutral value at $x = \bar{x}$ then the wavenumber K becomes complex for $x \neq \bar{x}$ so the wavy vortex mode undergoes spatial amplification or decay away from the neutral location.

We do not repeat the HL calculation in order to obtain specific values for K, Ω for a particular boundary layer flow. We believe that the major result of this paper is that the large amplitude states of HL are unstable in the thin shear layers which trap the vortices. The only available experimental results which give detailed results on the wavy mode structure do not give sufficient detail about the unperturbed boundary layer to enable us to calculate the relevant values of K and Ω, so below we discuss only the qualitative agreement between our theory and these experimental results. However, before we compare our results with experiments we shall first describe the eigenfunctions appropriate to (4.1).

These functions are shown in Figures 2, 3 and we point out their oscillatory nature for negative values of η. We note that $\eta \to -\infty$ corresponds to moving from IIa,b into the core region I. It is of course possible that other "lower order" modes with a less oscillatory nature might exist but, as stated already, only those corresponding to (4.1) were found.

Now let us turn to the physical implications and experimental relevance of our calculation. We first stress that the instability mechanism which we have described in detail for just the upper shear layer can also occur in the lower layer. The modes of instability of the shear layers are independent because they decay exponentially away from the center of the layers. Thus if we consider the frequency of the imposed wavy mode to be fixed then the layers will breakdown in the manner described at different downstream locations. Since the downstream velocity component of the basic state is largest in the upper shear layer it is to be expected that this layer will be the first to become unstable. Furthermore, since the wavenumbers $K(x)$ appropriate to a fixed frequency disturbances will be different, the modes propagates downstream with different wavespeeds. In fact initially, by which we mean close to the linear neutral position, the layers IIa,b coalesce so that if breakdown occurs close to this point then the structure in these layers will be very similar. Further downstream the upper layer moves into the free stream and the downstream velocity component tends to the free stream speed. The lower layer however approaches the boundary so that the fluid velocity there tends to zero. Thus, it follows that if the stationary Görtler vortices develop over a sufficiently long interval before breakdown then the upper layer wavy mode moves downstream with the free stream speed whilst the lower one has a much smaller propagation speed. It is interesting to note that in the apparently closely related Taylor vortex problem the corresponding breakdown is due to a single wavy mode whose presence is felt throughout the flow.

There have been many experimental investigations of the secondary instability of Görtler vortices; the reader is referred to the papers by for example Bippes and Görtler (1972), Wortmann (1969), and Bippes (1978) who all described the secondary mode as being locally periodic in x and t. However more recently Kohama (1987) and Peerhossaini and Wesfreid (1987) have given more details of the flow structure which exists when breakdown occurs. We first discuss the results of Peerhossaini and Wesfreid.

The boundary layer investigated by the latter authors was in the concave section of a curved channel. They found that when the secondary mode first appeared it was confined to a region at the top of the vortices. We interpret this as being due to the instability mechanism we have described being first operational in region IIa. Further downstream they reported a similar instability but this was localized near the wall. We interpret this instability as being due to the wavy vortex instability of region IIb. Both instabilities observed by Peerhossaini and Wesfreid had wavy vortex boundaries in the downstream direction, this is entirely consistent with the breakdown mechanism we have described in Section 3.

The experiments of Kohama were performed on the NASA laminar flow wing discussed by for example Pfenninger, Reed, and Dagenhart (1980). Kohama described only the breakdown in the upper part of the boundary layer but gave measurements of the wavespeeds at different downstream locations. In the laminar flow region of the wing the wavespeed was found to be about 0.45 of the free stream speed but this factor became about 0.99 when the flow was fully turbulent. Such a variation of wavespeed is predicted qualitatively by our theory since the layer IIa migrates from being somewhere in the middle of the boundary layer (more precisely at the position where Rayleigh's criterion for the pre-Görtler flow is most violated) to the edge as the vortices become fully nonlinear. The migration of this layer we believe accounts for the variation of wavespeed given by Kohama.

Finally we note that there are other instability mechanisms which could account for the appearance of three-dimensionality and time-dependence in the later stages of Görtler vortex development. The most likely other types of disturbances would be Rayleigh instabilities associated with the spanwise locally inflexional velocity profiles which certainly develop as the vortices develop in a steady manner. Secondly, there exists the possibility that Tollmien-Schlichting waves might cause the flow to become three-dimensional. However, it is not clear that these modes would lead to the wavy vortex boundaries which seem to be always observed when breakdown occurs. Nevertheless, it is likely that in some situations the Rayleigh and Tollmein-Schlichting modes might be important in the later stages of the transition process.

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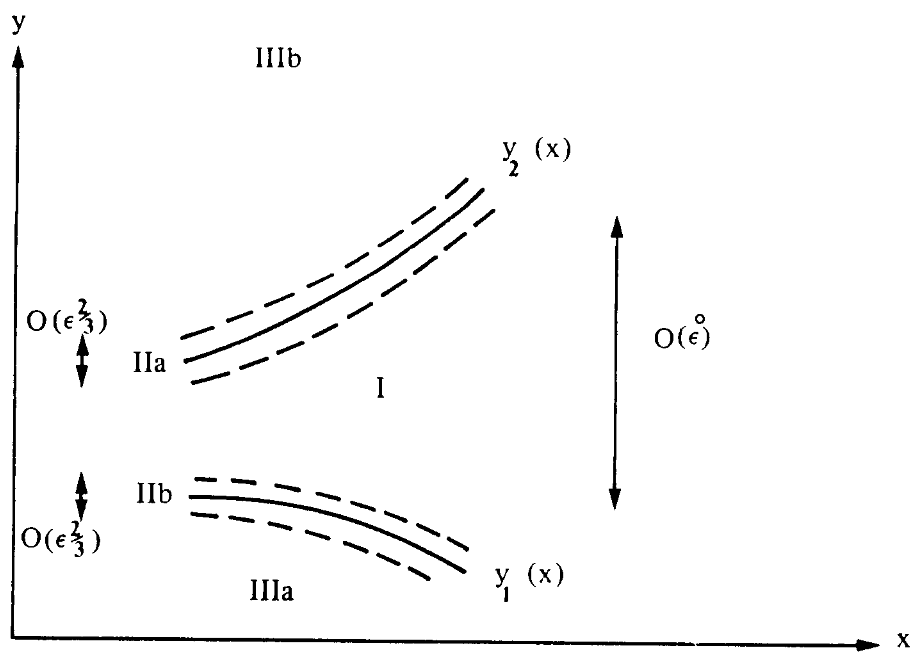
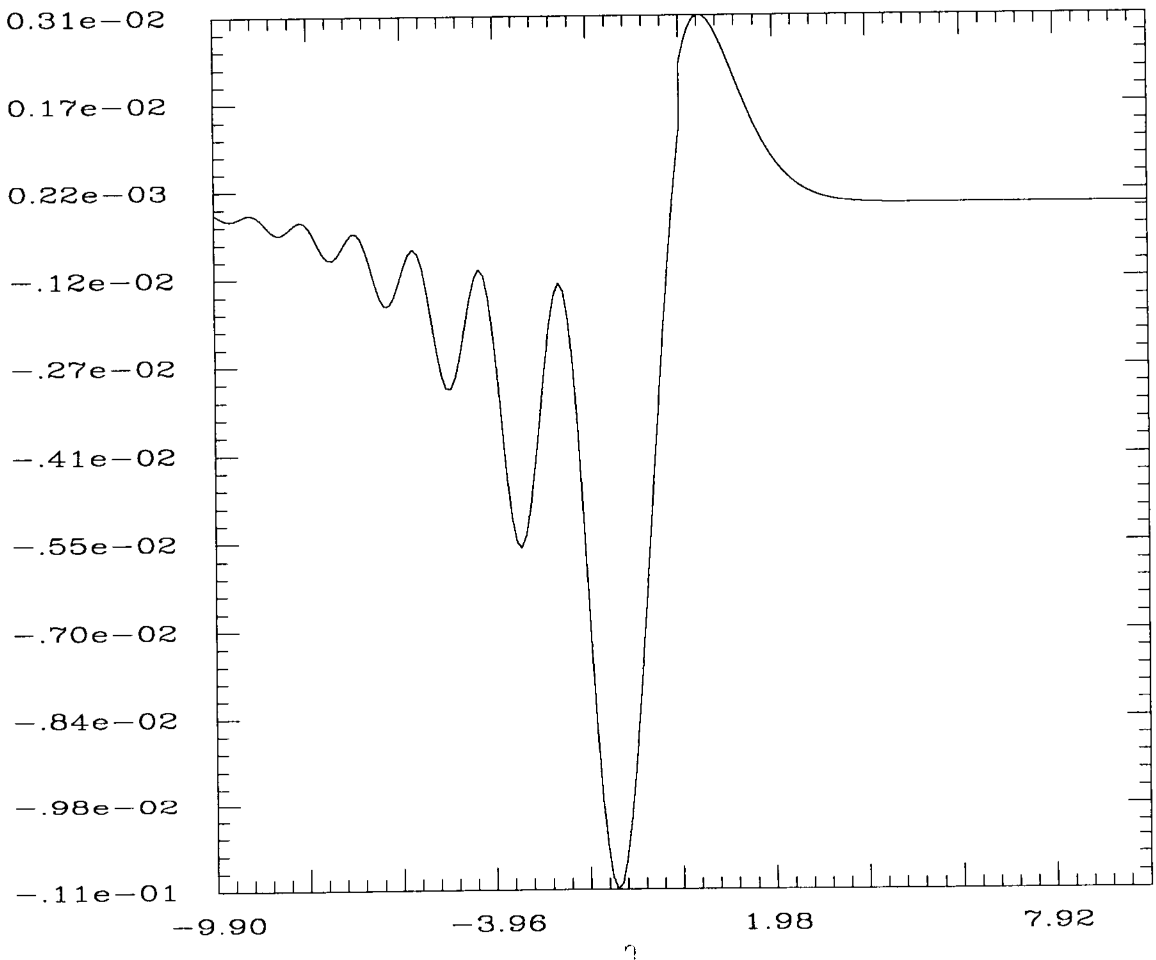
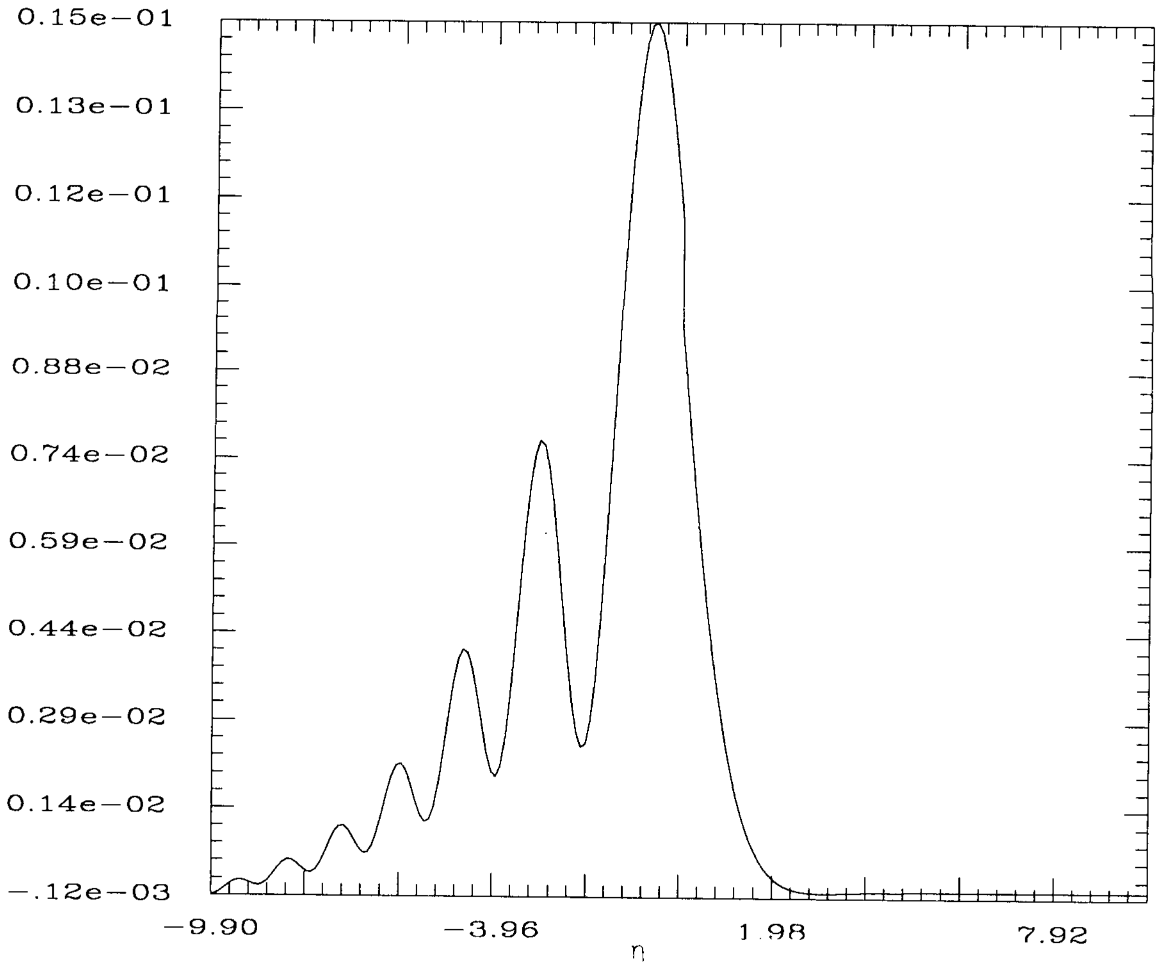


Figure 1. The different regions beyond the downstream position of neutral stability.



2(a)

Figure 2: The neutral eigenfunctions V_p and w_{mo} for $(\hat{K}_1,\hat{\Omega}) = (0.97,0.58)$ plotted against η: (a) $Re(V_p)$, (b) $Im(V_p)$, (c) $Re(w_{m0})$, (d) $Im(w_{m0})$.



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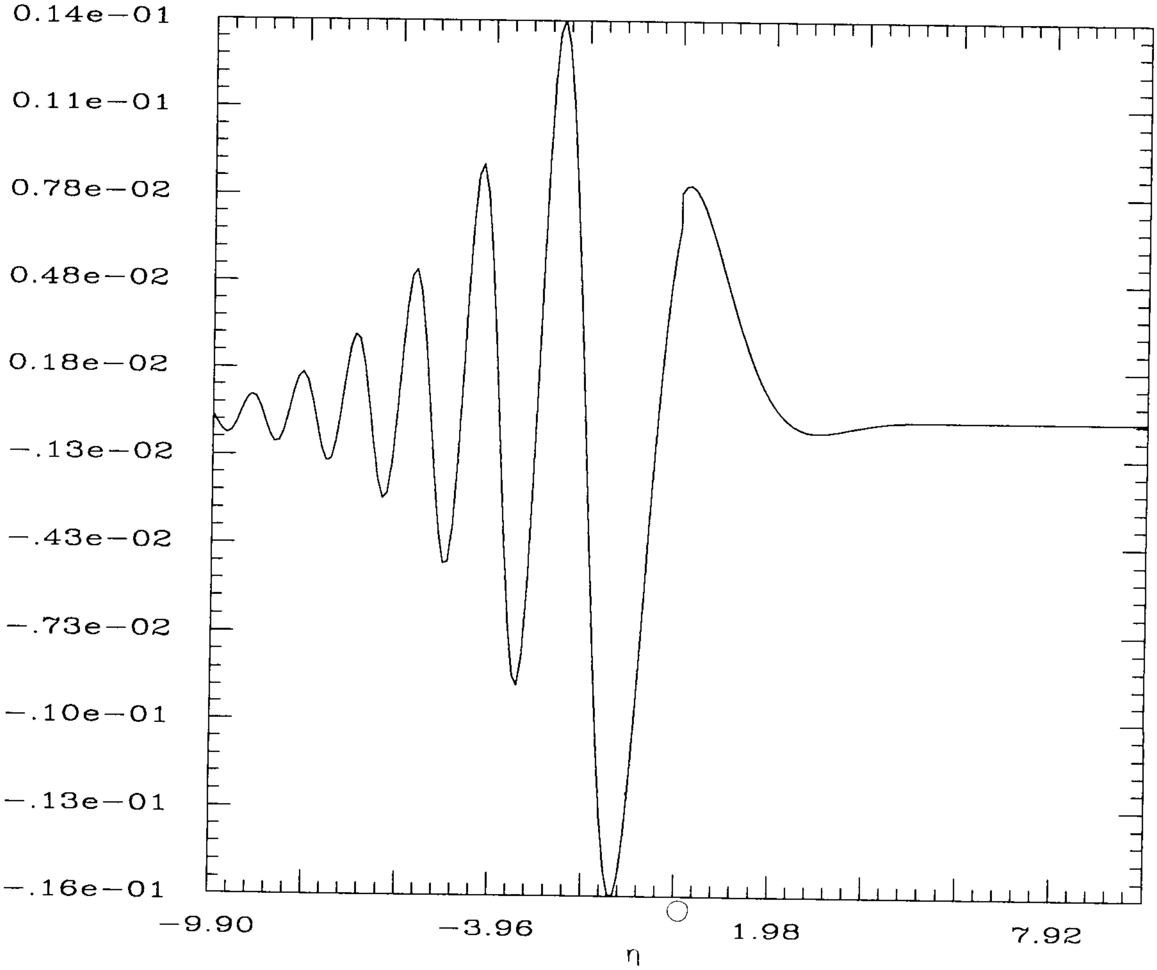
2(b)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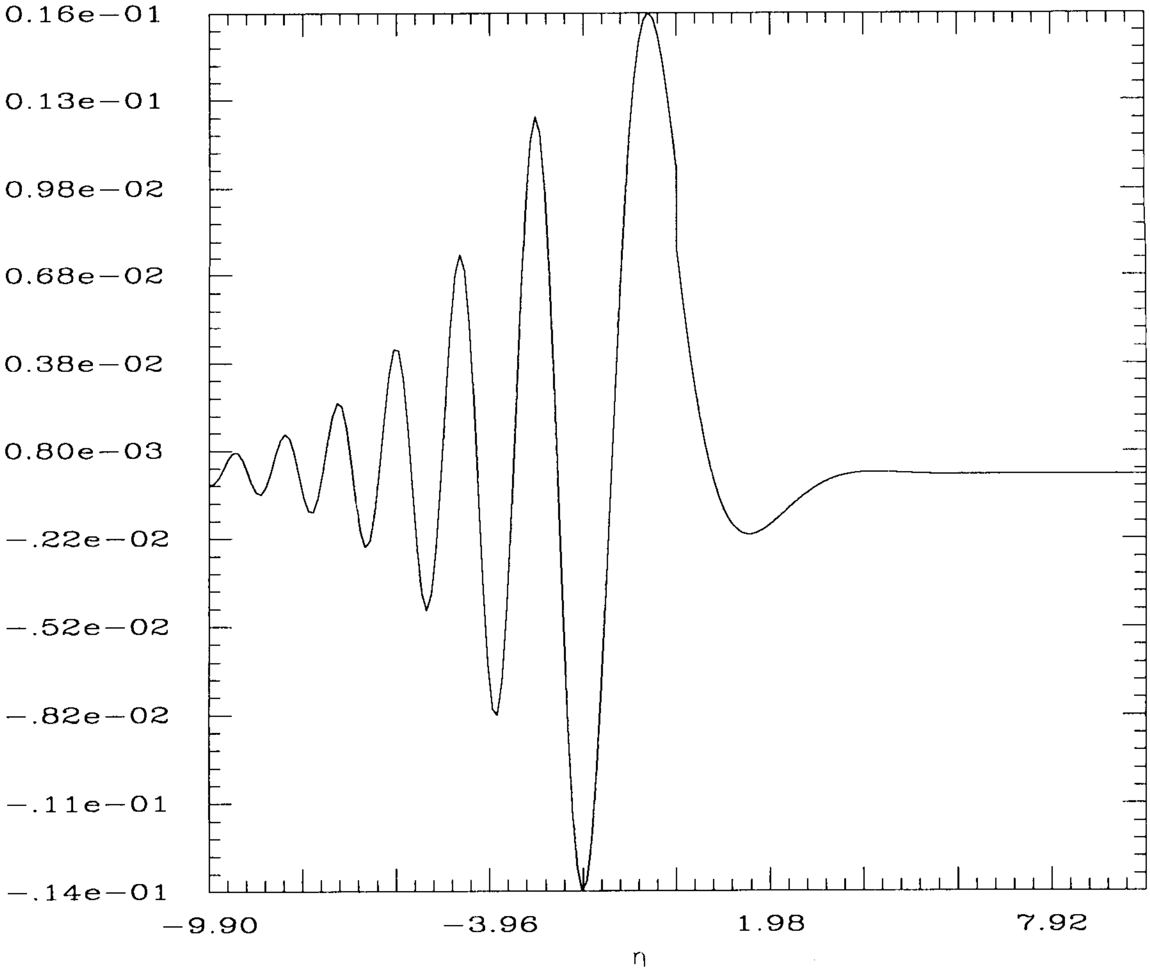
2(c)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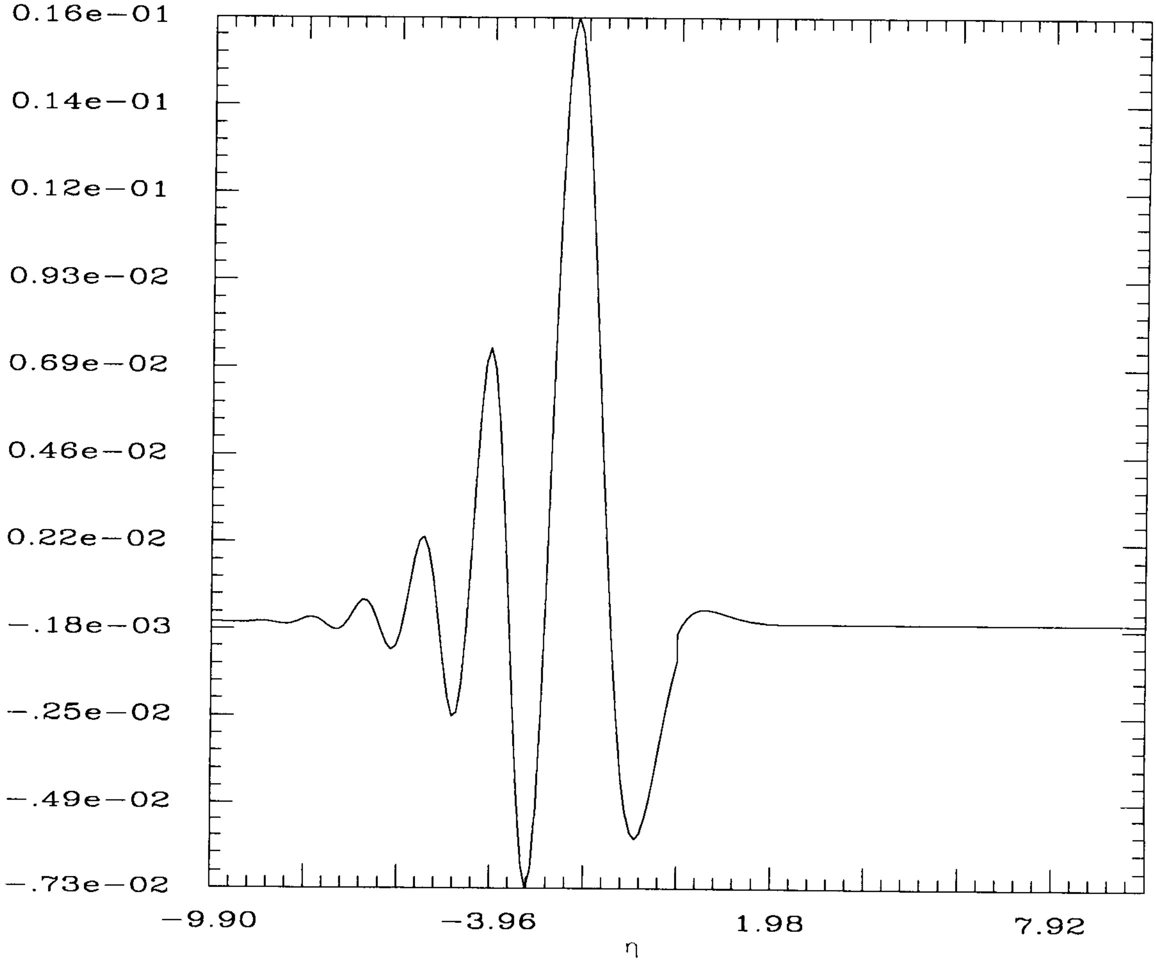
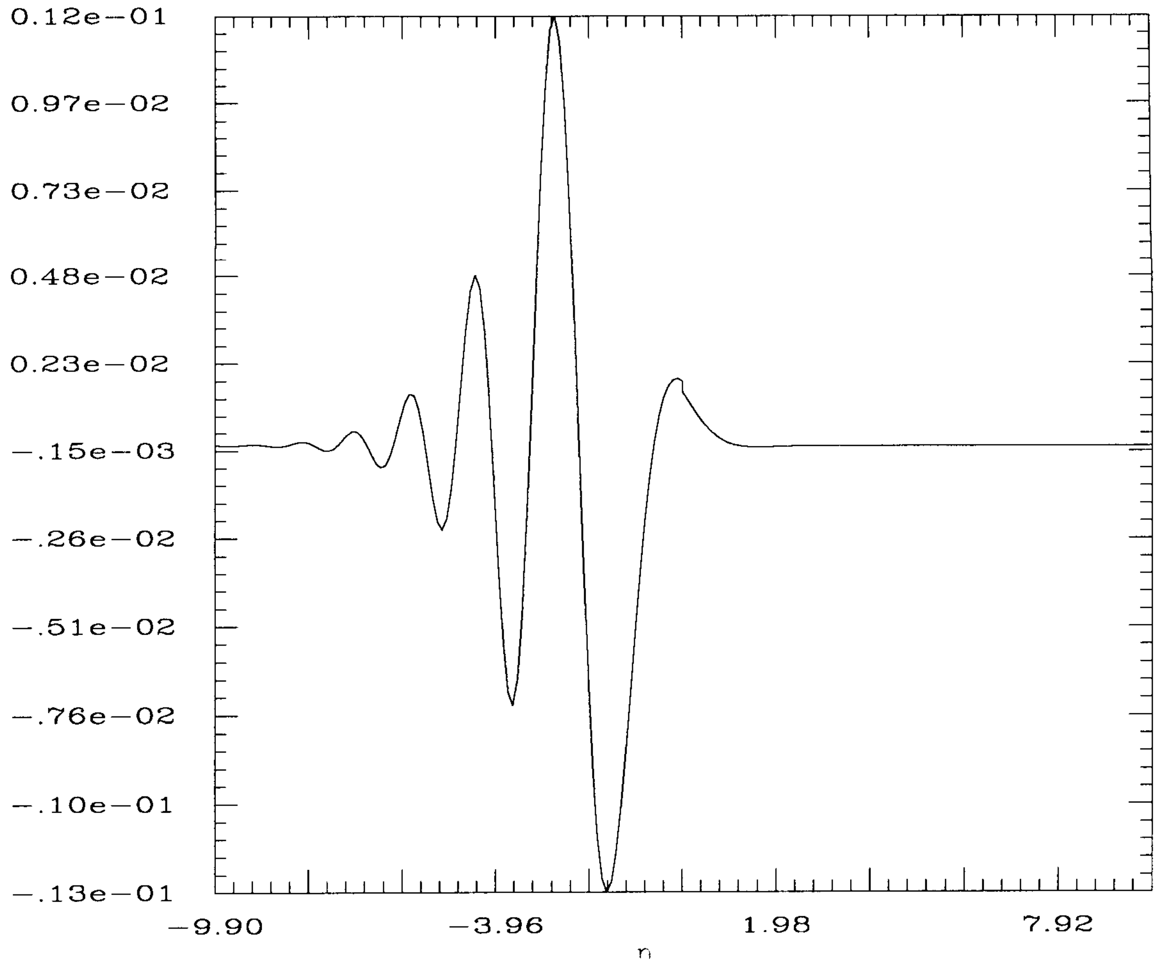
2(d)



3(a)

Figure 3. The neutral eigenfunctions V_p and w_{m0} for $(\hat{K}_1,\hat{\Omega})$ = (5.132,1.805) plotted aginst η: (a) $Re(V_p)$, (b) $Im(V_p)$, (c) $Re(w_{m0})$, (d) $Im(w_{m0})$.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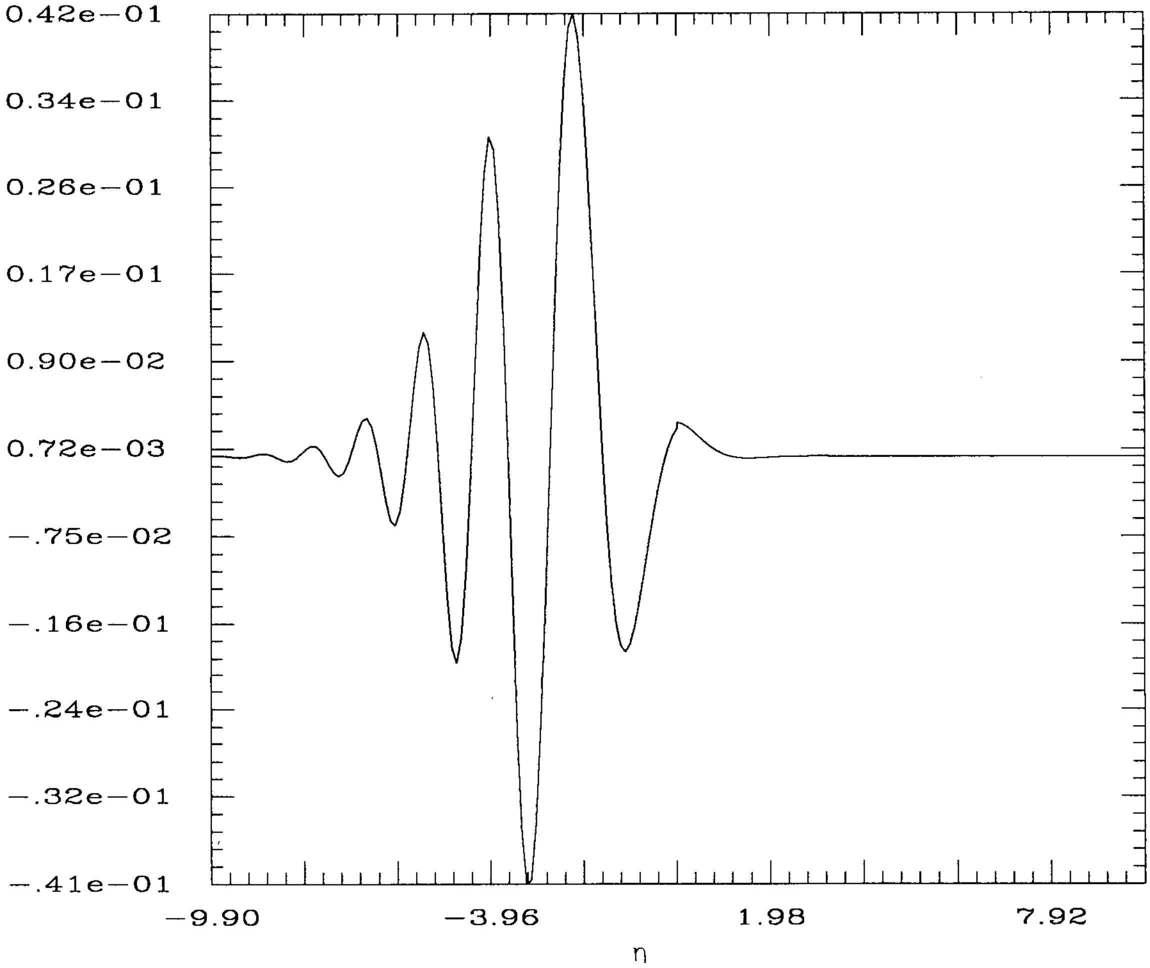
3(b)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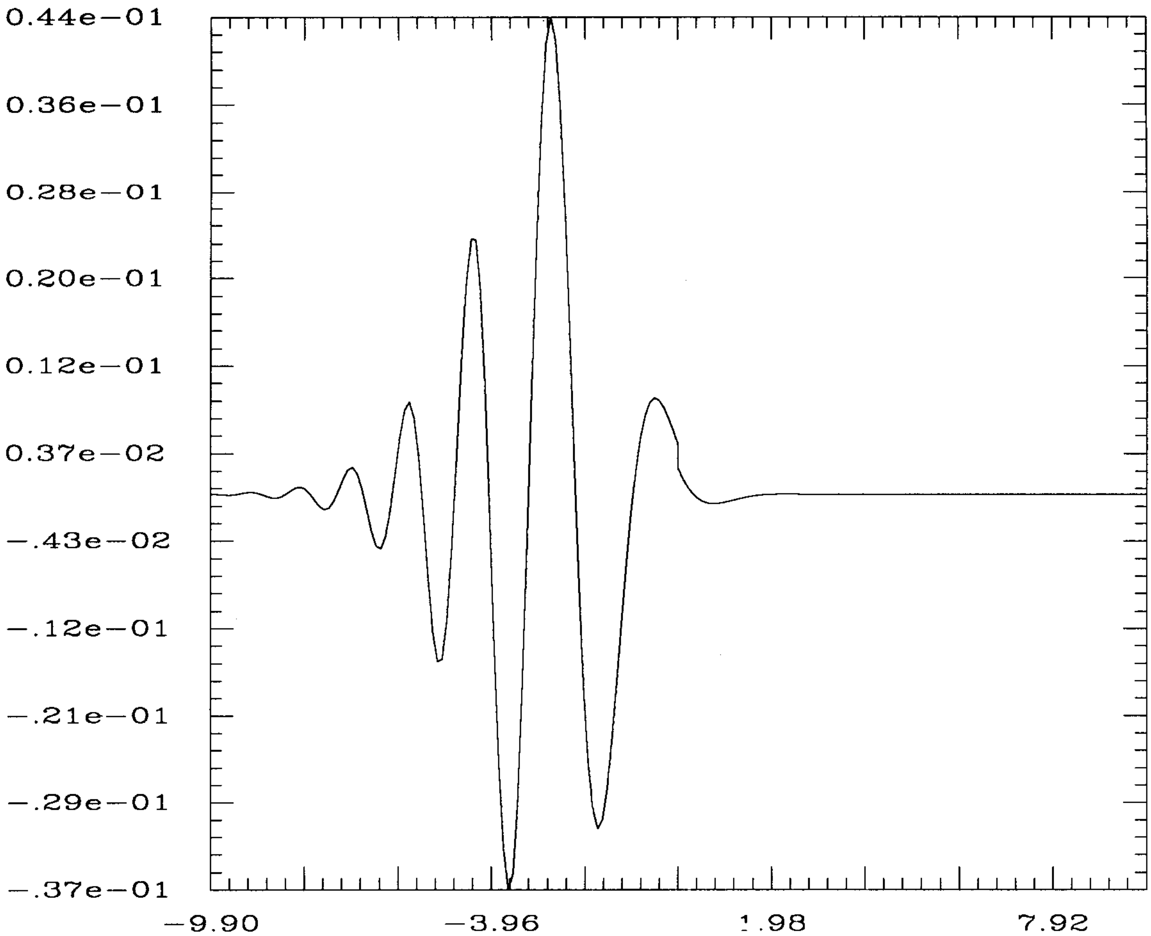
3(c)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3(d)

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16. Abstract

The instability of large amplitude Görtler vortices in a growing boundary layer is discussed in the fully nonlinear regime. It is shown that a three-dimensional breakdown to a flow with wavy vortex boundaries similar to that which occurs in the Taylor vortex problem takes place. However, the instability is confined to the thin shear layers which were shown by Hall and Lakin (1987) to trap the region of vortex activity. The disturbance eigenfunctions decay exponentially away from the center of these layers so that the upper and lower shear layers can support independent modes of instability. The structure of the instability, in particular its location and speed of downstream propagation, is found to be entirely consistent with recent experimental results. Furthermore, it is shown that the upper and lower layers support wavy vortex instabilities with quite different frequencies. This result is again consistent with the available experimental observations.

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